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Field-induced incommensurate order in frustrated spin ladder

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Abstract
In the magnetization process of the $S = 1/2$ spin ladder system there are two critical magnetic fields H_{c1} and H_{c2}. The spin gap vanishes at H_{c1} and the magnetization is saturated at H_{c2}. Usually the commensurate antiferromagnetic spin correlation perpendicular to the external field H is dominant and the canted Néel order would be stabilized by interladder interaction for $H_{c1} < H < H_{c2}$. The present theoretical study based on numerical diagonalization indicated the following interesting results. In the presence of the next-nearest-neighbour interaction, the incommensurate spin correlation parallel to H would be possibly stronger than the commensurate one for some intermediate external field. As a result, with interladder interaction, another field-induced transition would occur from the usual canted Néel order to the incommensurate one. The new field-induced transition is based on the so-called η-inversion, namely a change from $\eta^z > \eta^x$ to $\eta^z < \eta^x$, where η^z and η^x are the critical exponents of the parallel and perpendicular spin correlation functions, respectively. Some phase diagrams including the η-inversion and the magnetization plateau are presented.

1. Introduction

The frustration of the antiferromagnetic exchange interaction enhances the quantum fluctuation in low-dimensional magnets and results in various interesting quantum critical phenomena. The previous numerical study [1, 2] on the spin ladder system revealed that with sufficiently large frustration due to the next-nearest-neighbour (diagonal) interaction, a field-induced spin gap would appear at a half of the saturation magnetization, namely the 1/2 magnetization plateau [3–6] would be observed.

In the present paper, as another new interesting phenomenon caused by the frustration, we propose a field-induced incommensurate order. It can be realized when the incommensurate

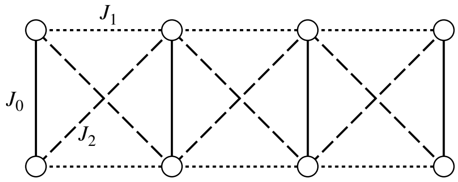

Figure 1. Schematic figure of the present model Hamiltonian (1).

spin correlation parallel to the external magnetic field is larger than the perpendicular antiferromagnetic one, the so called η inversion indicated later, in the presence of interladder interaction. Such a possibility was also discussed for the frustrated bond-alternating chain [7, 8]. We present some phase diagrams including a magnetization plateau, because the η inversion should be accompanied by the plateau.

2. Model of frustrated spin ladder

We consider the $S = 1/2$ uniform antiferromagnetic spin ladder with the next-nearest-neighbour exchange interaction as shown in figure 1. Our Hamiltonian is described by

$$\hat{H} = J_0 \sum_{j=1}^{N} \boldsymbol{S}_{j,1} \cdot \boldsymbol{S}_{j,2} + J_1 \sum_{j=1}^{N} (\boldsymbol{S}_{j,1} \cdot \boldsymbol{S}_{j+1,1} + \boldsymbol{S}_{j,2} \cdot \boldsymbol{S}_{j+1,2}) + J_2 \sum_{j=1}^{N} (\boldsymbol{S}_{j,1} \cdot \boldsymbol{S}_{j+1,2} + \boldsymbol{S}_{j,2} \cdot \boldsymbol{S}_{j+1,1}) - H \sum_{j=1}^{N} \left(S_{j,1}^z + S_{j,2}^z \right) \tag{1}$$

where J_0 and J_1 are the rung and leg exchange constants, respectively. We set $J_0 = 1$ throughout this paper. J_2 is the next-nearest-neighbour exchange constant. The applied magnetic field is denoted by H. The magnetization of the bulk system is defined as $m = M/L$, where $M \equiv \langle \sum_j (S_{1,j}^z + S_{2,j}^z) \rangle$.

3. η inversion

Applying an external magnetic field to the gapped spin systems, the gap vanishes at some critical value H_{c1} and the gapless Tomonaga–Luttinger liquid phase is realized for $H > H_{c1}$ [9, 10]. It is characterized by the power-law decay of the spin correlation functions,

$$\langle S_0^x S_r^x \rangle \sim (-1)^r r^{-\eta^x}, \tag{2}$$

$$\langle S_0^z S_r^z \rangle \sim \cos(2k_F r) r^{-\eta^z}, \tag{3}$$

where k_F is the Fermi momentum of the Tomonaga–Luttinger liquid, which is related to the magnetization as $k_F = \pi m$. The first spin correlation function (2) describes an ordinary antiferromagnetic correlation perpendicular to H and it would be the long-range canted Néel order with appropriate interladder interaction. The second one (2) corresponds to an incommensurate spin correlation parallel to H. In usual antiferromagnets, $\eta^x < \eta^z$ is satisfied and the transverse antiferromagnetic correlation is dominant, irrespective of H. As a result, the canted Néel order would be observed with interladder interaction. In the presence of some frustrated interaction, however, recent numerical studies revealed that some one-dimensional systems can exhibit $\eta^x > \eta^z$ in some intermediate H close to $m = m_s/2$, where m_s is

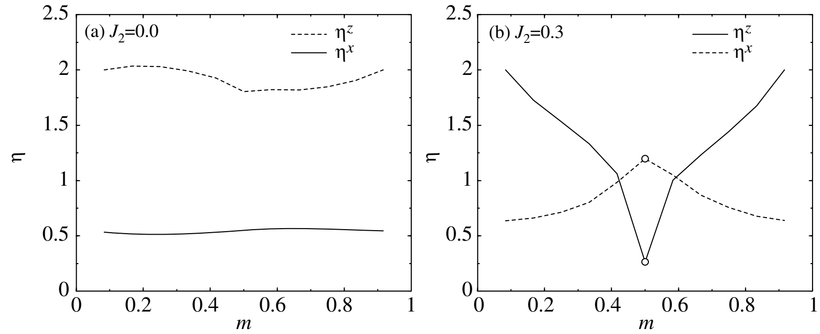

Figure 2. The exponents $\eta_\perp$ and $\eta_\parallel$ versus m for (a) $J_1 = 1.0$ and (b) $J_1 = 0.5$. Open circles indicate the values just at $m = 1/2$ estimated by use of (3) for the finite system ($L = 12$). Although the spin correlation functions should decay exponentially just at the half plateau for $L \to \infty$, these calculated values are expected to be good approximations of $\lim_{m\to 1/2-} \eta = \lim_{m\to 1/2+} \eta$.

the saturation magnetization [7, 11]. Namely, the incommensurate spin correlation parallel to H can be dominant. In this case, the incommensurate order can occur in the quasi-one-dimensional frustrated systems [7]. The realization of $\eta^x > \eta^z$ is called 'η inversion'. Since the universal relation $\eta^x\eta^z = 1$ should hold for the Tomonaga–Luttinger liquid, the boundary value of H for the η inversion should be the field for which $\eta^x = \eta^z = 1$. On the other hand, the previous study to indicate the magnetization plateau in the present model (1) suggested [1, 2] that the critical values of J_2 and H should be given by $\eta^x = \eta^z = 1$. It implies that the present system should exhibit the η inversion at least in the parameter region where the half plateau appears. Since the plateau can appear for $J_1 < J_0\,(=1)$, we fixed J_1 to 0.5 as a typical value throughout this paper.

The critical exponents η^x and η^z can be calculated numerically by the size-scaling forms,

$$\Delta_1 \sim v_s \eta^x \frac{1}{L}, \tag{4}$$

$$\Delta_{2k_F} \sim v_s \eta^z \frac{1}{L}, \tag{5}$$

where Δ_1 is the spin gap, Δ_{2k_F} is the $2k_F$ excitation gap and v_s is the sound velocity [10]. Neglecting the size correction, the calculated η^x and η^z for $L = 12$ are plotted versus the magnetization m in figures 2(a) for $J_2 = 0$ and (b) for $J_2 = 0.3$, respectively. They show that for sufficiently large J_2 it holds that $\eta^x > \eta^z$, namely η inversion occurs around $m \sim 1/2$, while it does not for $J_2 = 0$. With the interchain interaction, the system would exhibit the long-range order corresponding to the dominant spin correlation, which has the smaller η, in the thermodynamic limit.

4. Phase diagram

As shown in the previous section, the η inversion should occur around $m \sim 1/2$. Since the condition for existence of the half plateau is $\eta^x = \eta^z = 1$, the η inversion should be accompanied with the plateau. Thus we assume that the half plateau exists for $J_2 > J_{2c}$, which was given in the previous work [2].

In order to present the phase diagram in the m–J_2 plane, we estimate the critical value of m satisfying $\eta^z(m) = 1$, because the finite-size correction is smaller for η^z than η^x [13]. The function $\eta^z(m)$ of finite systems is a discrete one calculated only for $m = n/L$, where n is

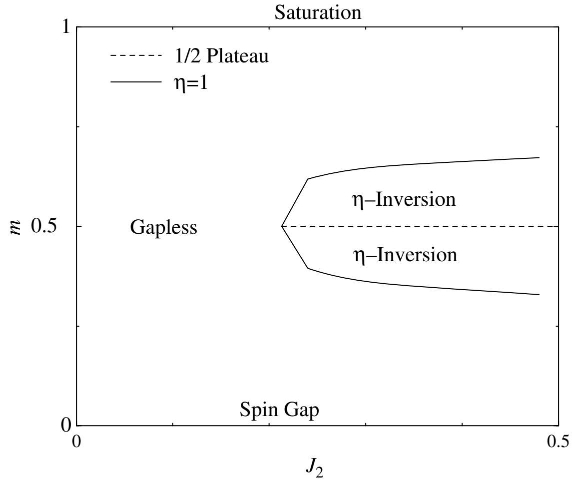

Figure 3. m–J_2 phase diagram for $J_1 = 0.5$.

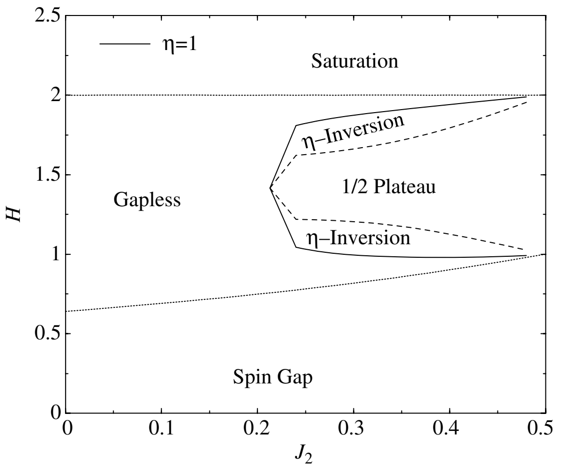

Figure 4. H–J_2 phase diagram for $J_1 = 0.5$.

integer. Thus we perform a suitable interpolation to determine the point for $\eta^z(m) = 1$. The results of the phase diagram calculated for $L = 12$ are shown in figure 3 for $J_1 = 0.5$. It is shown that the η inversion region appears around $m \sim 1/2$ and the half plateau divides the region into two parts. The left edge of the plateau line was determined by the previous level spectroscopy analysis [2].

The H–J_2 phase diagram would be very useful to consider some realistic situations observed in experiments. A similar interpolation technique for the external magnetic field H based on the finite-cluster calculation for $L = 12$ yields the H–J_2 phase diagram in figure 4 for $J_1 = 0.5$. The half plateau is indicated by the region surrounded by dashed curves and the η inversion appears inside solid curves except for the plateau. H_{c1} and H_{c2} are also shown as dotted curves.

5. Summary

The $S = 1/2$ frustrated spin ladder system with the next-nearest-neighbour exchange interaction in magnetic field is investigated by the numerical exact diagonalization of a finite cluster. It indicates that, instead of the usual transverse antiferromagnetic spin correlation, the incommensurate one parallel to H can be dominant (η inversion) around half of the saturation magnetization for sufficiently large J_2. The critical value J_{2c} for the appearance of the half magnetization plateau corresponds to the lower boundary of the η inversion. Several phase diagrams including the plateau phase and the η inversion regions are presented. With appropriate interchain interaction, the incommensurate long-range order would appear in the η inversion region. A DMRG analysis with a mean field approximation for interchain interaction [7, 12] suggested a first-order phase transition between the field-induced incommensurate and usual antiferromagnetic orders in the $S = 1/2$ frustrated bond-alternating chain. More detailed analyses, however, would possibly reveal the coexistence of the two orders at lower temperature phases. A field theoretical study [14] would indicate that these two ordered phases are results coming from the Bose–Einstein condensation of two different components, and the coexisting phase corresponds to the supersolid phase observed in ^{4}He [15]. It would be quite interesting to discover such a new phase, for example in the cuprate spin ladder system $Sr_{14}Cu_{24}O_{41}$ [16].

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